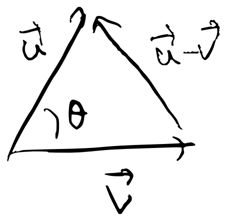
In this short note we explain why

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta.$$

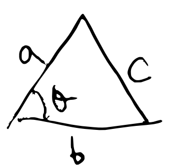
Note that $|\vec{u}-\vec{v}|^2 = (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$$= |\vec{u}|^2 - 2\vec{u}\cdot\vec{v} + |\vec{v}|^2$$



So, in particular $|\vec{u}-\vec{v}|$ is the length of the third side of the triangle formed by $\vec{u}$ & $\vec{v}$.

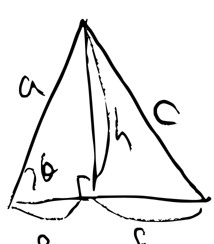
Now we have the second law of cosines: for a triangle



, $c^2 = a^2 + b^2 - 2ab\cos\theta$.

From this, we get $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$.

Recall how to deduce the second law of cosines:



, then

- $h = a\sin\theta$
- $e = a\cos\theta$
- $f = b - e = b - a\cos\theta$
- $c^2 = h^2 + f^2 = a^2\sin^2\theta + (b - a\cos\theta)^2$

$$= a^2\sin^2\theta + b^2 + a^2\cos^2\theta - 2ab\cos\theta$$
$$= a^2 + b^2 - 2ab\cos\theta.$$